WELCOME

Convergent Sequence : A Geometrical Approach

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Outline

Introduction to Sets

✍ Forget everything you know about numbers.

Introduction to Sets

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- In fact, forget you even know what a number is.

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- In fact, forget you even know what a number is.
- This is where mathematics starts.
- Instead of math with numbers, we will now think about math with "things".

Introduction to Sets

For Example

The items you wear: shoes, socks, hat, shirt, pants, and so on.

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Types of fingers. This set includes index, middle, ring, and pinky.

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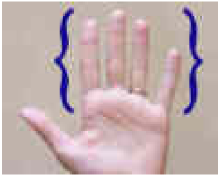

Introduction to Sets

For Example

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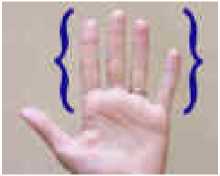

So it is just things grouped together with a certain property in common.

Introduction to Sets

What is set ?

Well, simply put, it's **a collection**.

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Definition

A set is a collection of well defined objects or things.

Introduction to Sets

Notations

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- If x is a not the member of the set S, then it is written as $x \notin S$ and read as x does not belong to S.

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Example

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$a \in V$, $i \in V$ but $b \notin V$

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Is it a set ?

No, because here brilliant is not defined.

Sets

$\mathbb{N}$ - Natural Numbers $(1, 2, 3, 4, \cdots)$

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$\mathbb{R}$ - Set of Real Numbers

Graphical View

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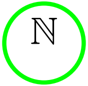
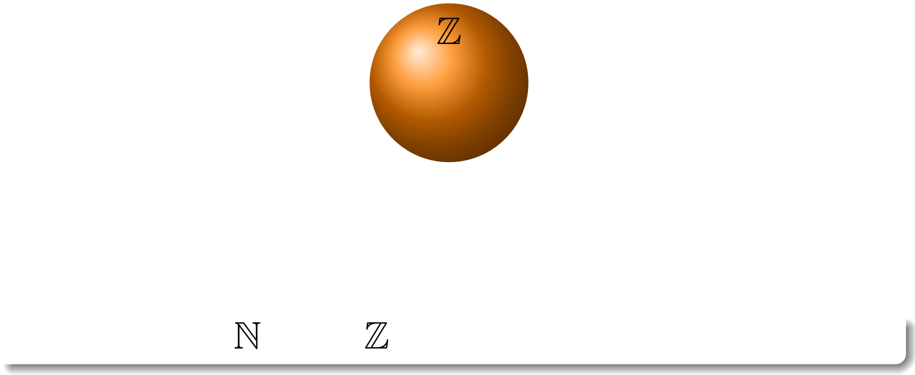
$\mathbb{N}$

Graphical View



$\mathbb{N}$

Graphical View



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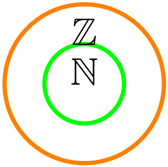
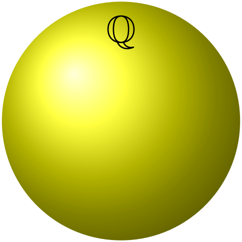


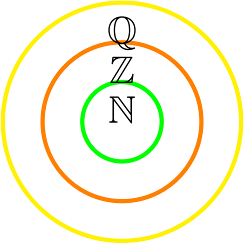
$\mathbb{N} \subset \mathbb{Z}$

Graphical View



$\mathbb{N} \subset \mathbb{Z} \quad \mathbb{Q}$

Graphical View



$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q}$$

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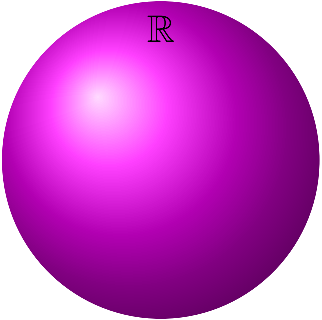

$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \quad \mathbb{R}$

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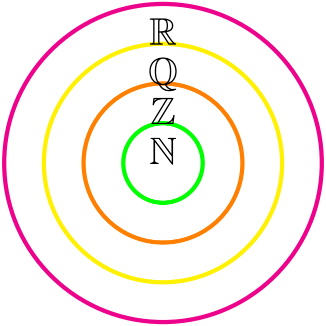

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Function

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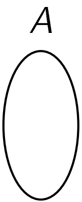
- In mathematically written as $f : A \to B$ defined by $f(a) = b$ for all $a \in A$.



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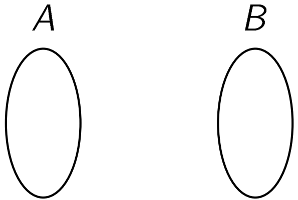

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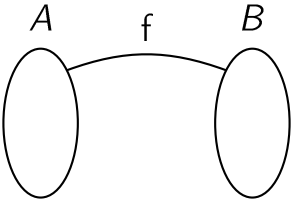

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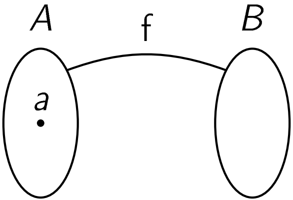

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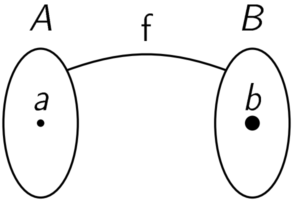

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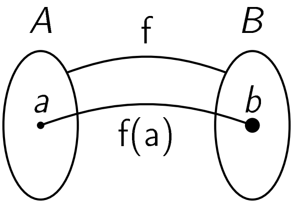

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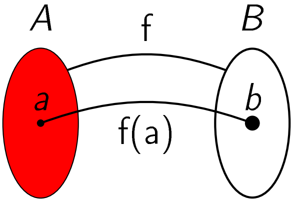

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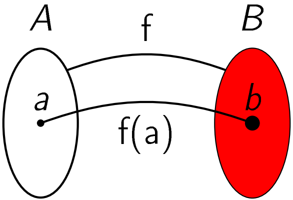

- Consider the function $f : A \to B$ by $f(a) = b$
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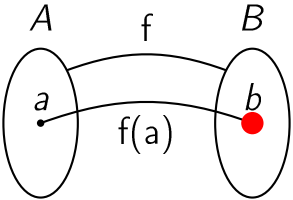

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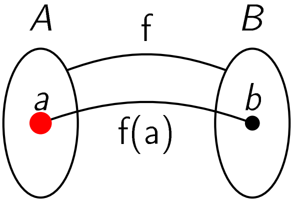

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- The element $a \in A$ is called the pre-image of b under f.

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Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$.

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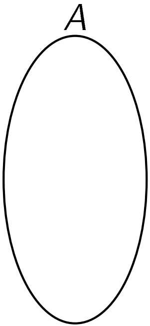
- Any negative real number x does not have a pre-image under f
- Example -9 has no pre-image under f
- So range set of this function is $\mathbb{R}^+ \cup \{0\}$.

Graphical



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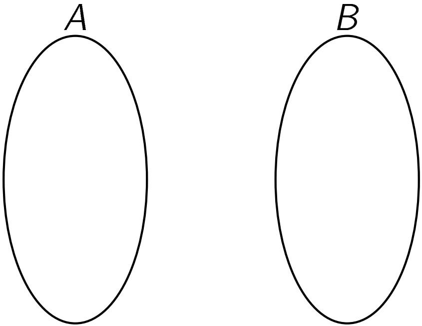

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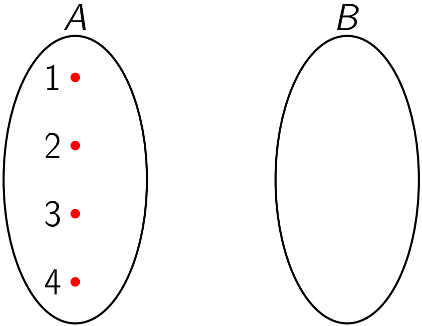

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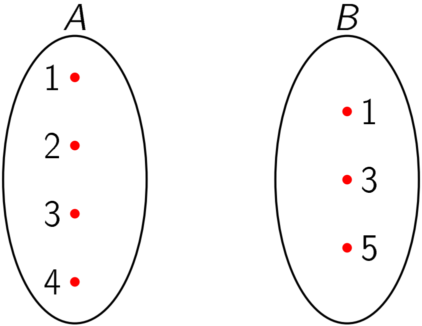

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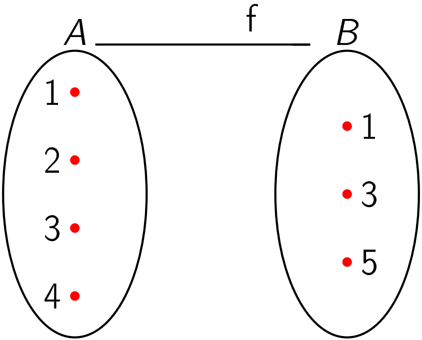

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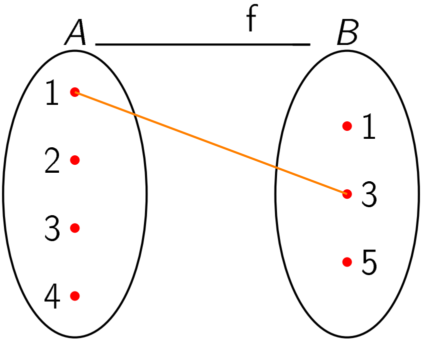

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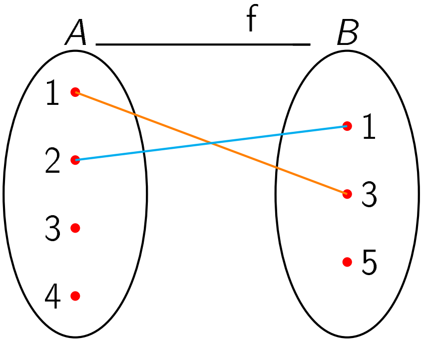

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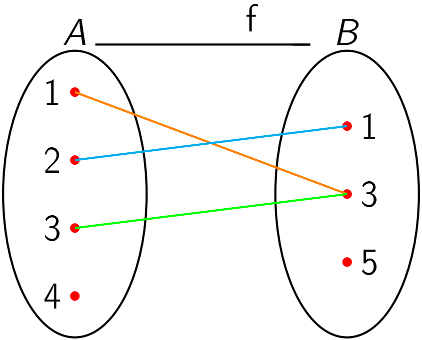

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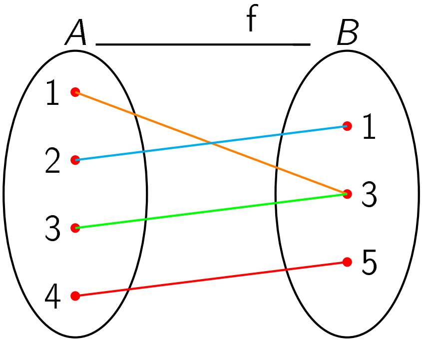

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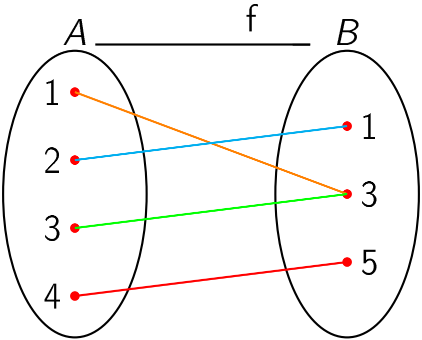
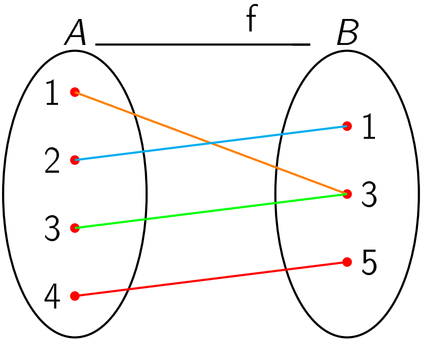


Is it function ?

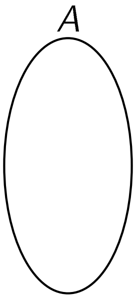
Graphical



Is it function ?

Yes

Graphical



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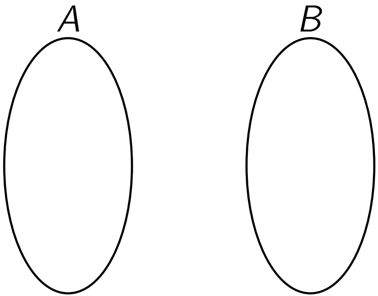

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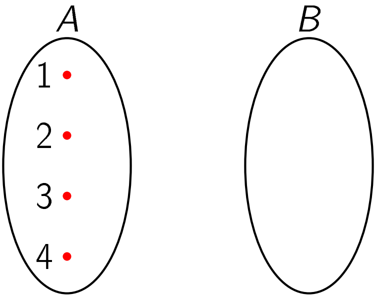

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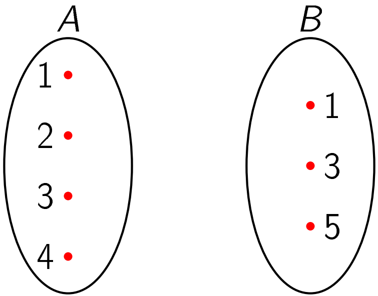

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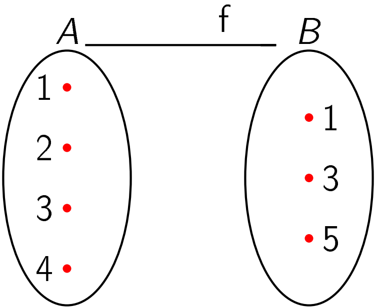

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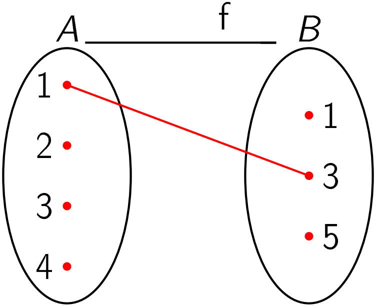

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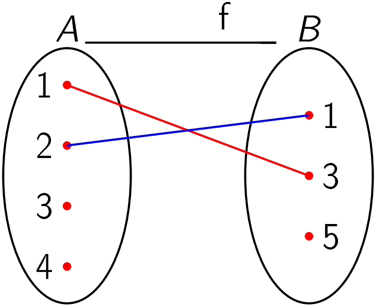

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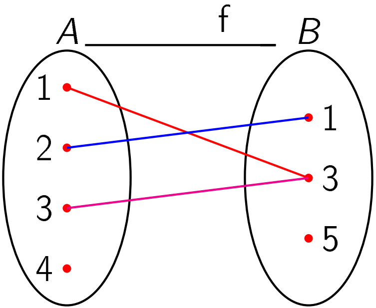

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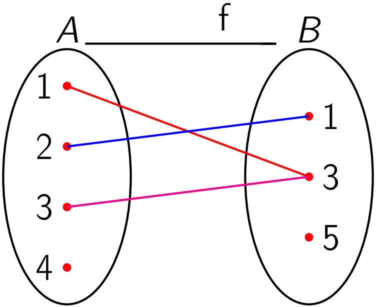

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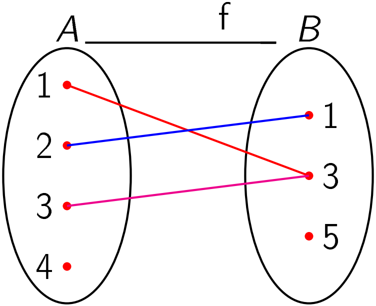
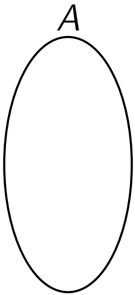


Is it function ?

No

Graphical



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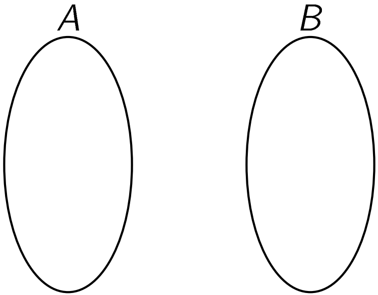

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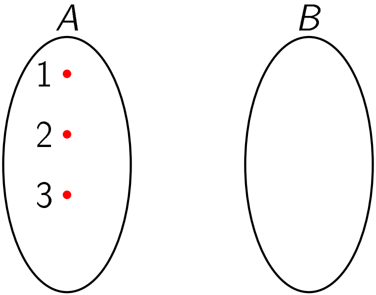

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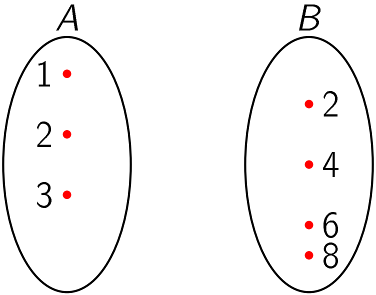

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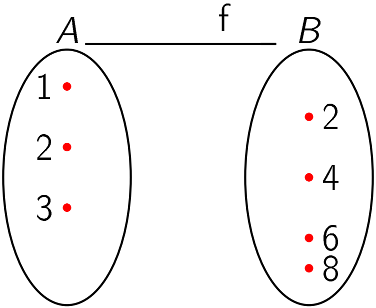

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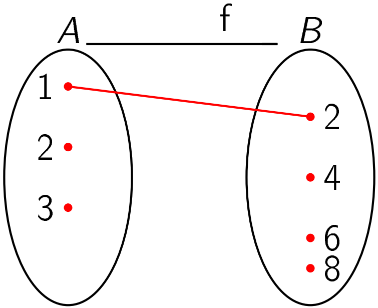

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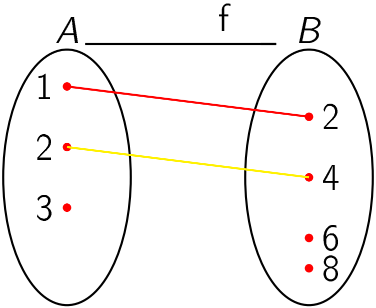

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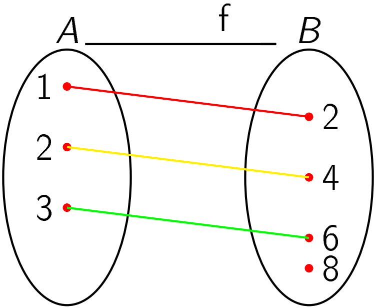

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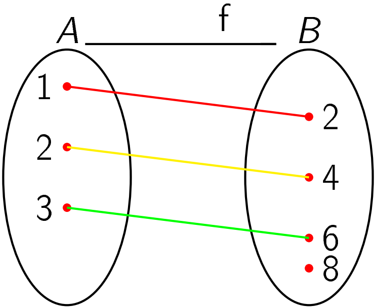
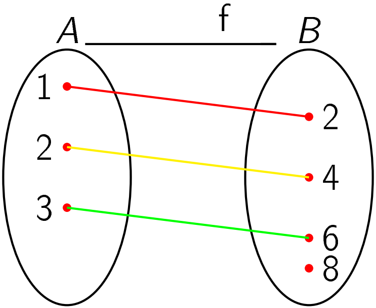


Is it function ? If it is, what type is it?

Graphical



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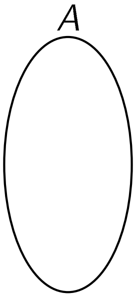
One - to - one (or) Injective

Graphical



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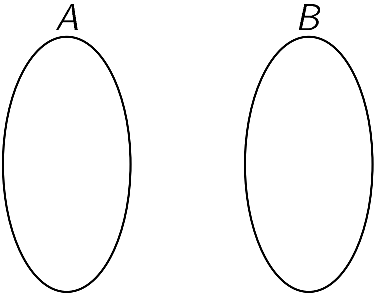

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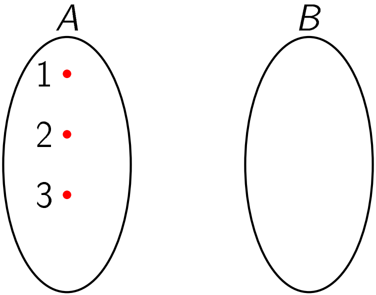

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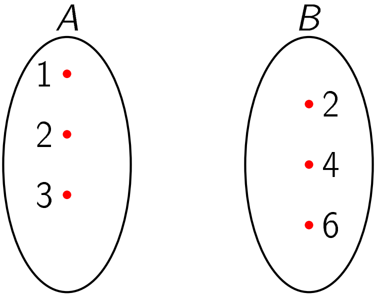

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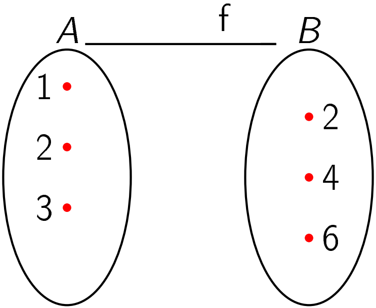

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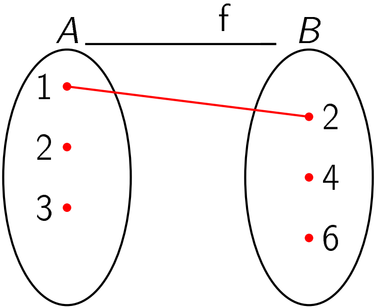

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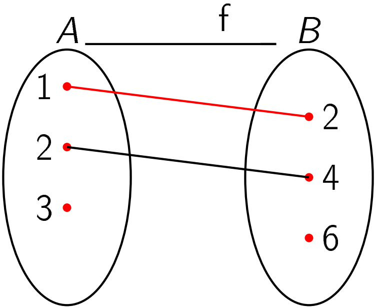

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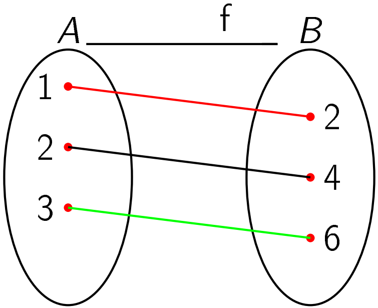

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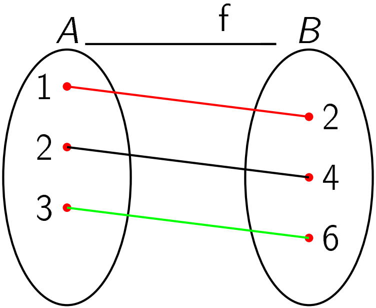

Is it function ? If it is, what type is it?

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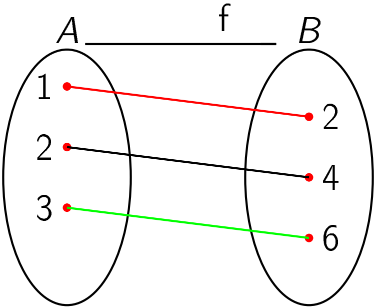

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Onto (or) Surjective

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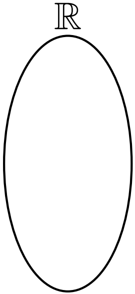

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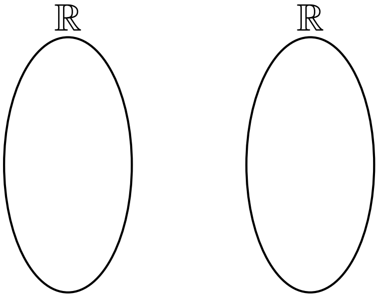

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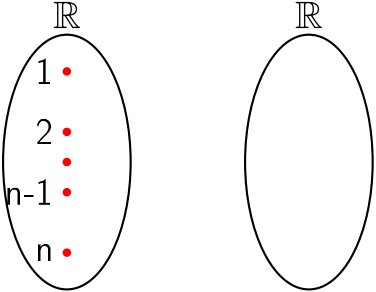

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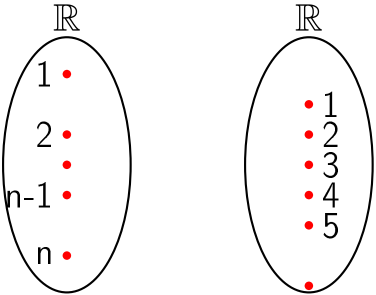

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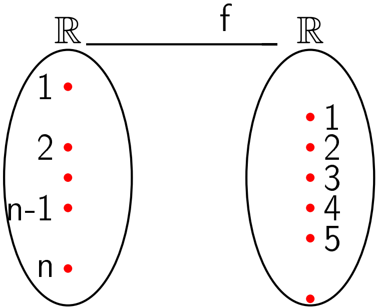

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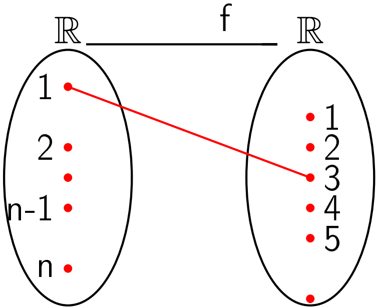

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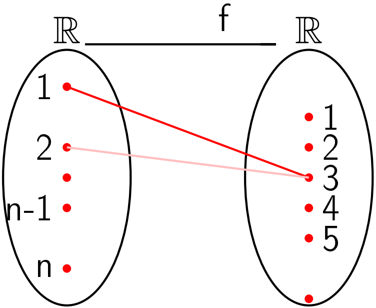

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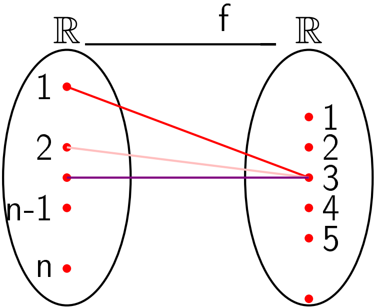

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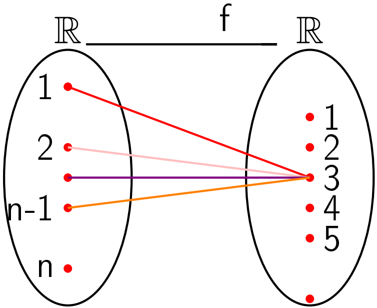

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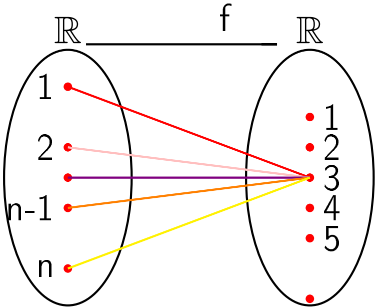

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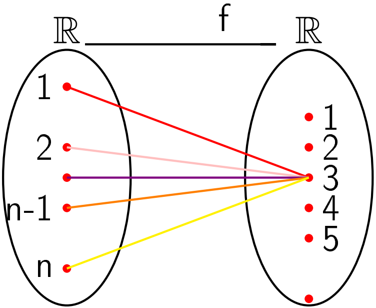

Constant Function

$f : \mathbb{N} \to \mathbb{N}$ defined by $f(x) = 3$ is called a constant function. The range of f is 3.

Introducing Sequence

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- If terms are next to each other they are referred to as consecutive terms.
- When we write out sequences, consecutive terms are usually separated by commas.

Example

Consider the following collection of real numbers given by

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \cdots$$

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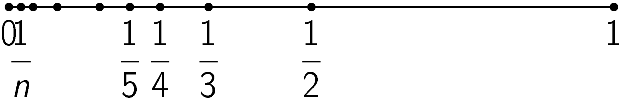

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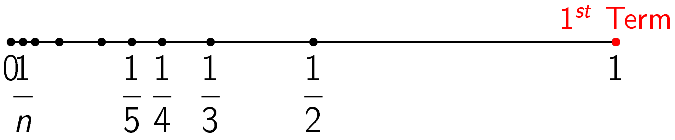

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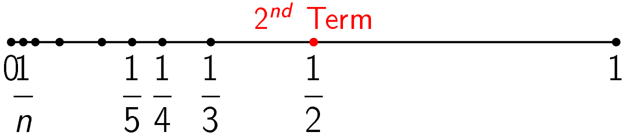

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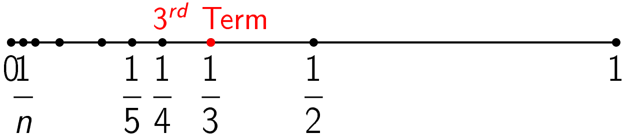

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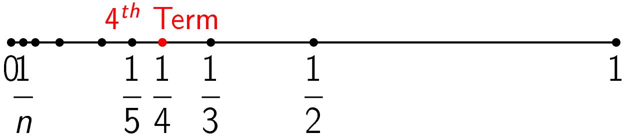

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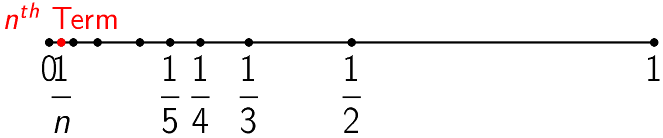

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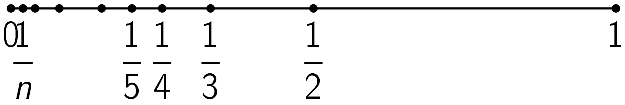

This is an example of sequence of real numbers.

Sequence is a function whose domain is the set of natural numbers.

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Definition

Let $f : \mathbb{N} \to \mathbb{R}$ be a function and $f(n) = a_n$. Then $a_1, a_2, a_3, \cdots, a_n, \cdots$, is called the sequence in $\mathbb{R}$ determined by the function f and is denoted by $\{a_n\}$, a_n is called the n^{th} term of the sequence.

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$$-1, 1$$

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Before giving the formal definition of convergence of a sequence, let us take a look at the behaviour of the sequences in the above examples.

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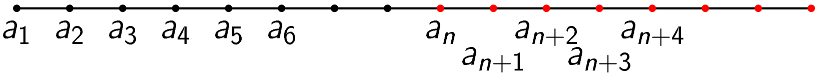
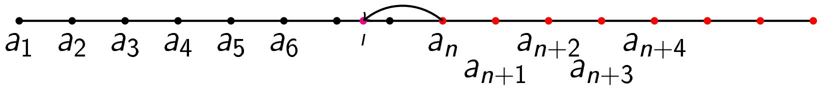
Convergence of a Sequence

We say that a sequence (x_n) converges if there exists $x_0 \in \mathbb{R}$ such that for every $\epsilon > 0$, there exists a positive integer N (depending on ϵ) such that $x_n \in (x_0 - \epsilon, x_0 + \epsilon)$ for all $n \geq N$.



Definition

Let $\{a_n\}$ be a sequence of real numbers.



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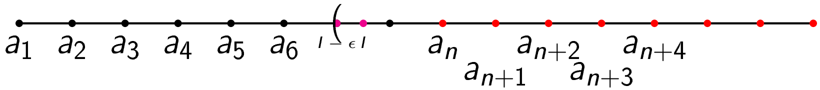
Let $\{a_n\}$ be a sequence of real numbers. $\{a_n\} \to l$



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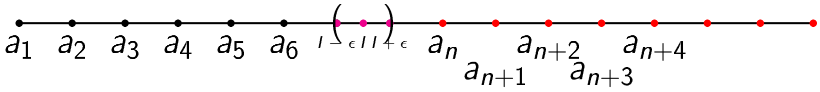

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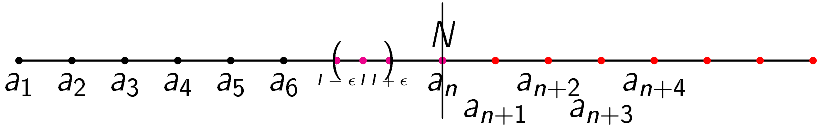

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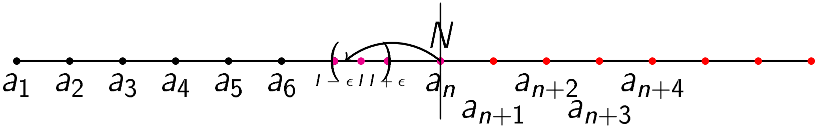

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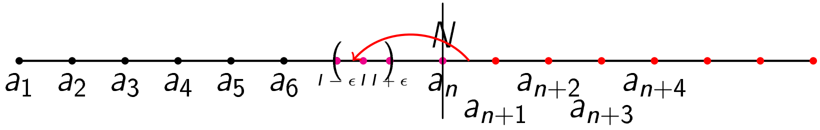

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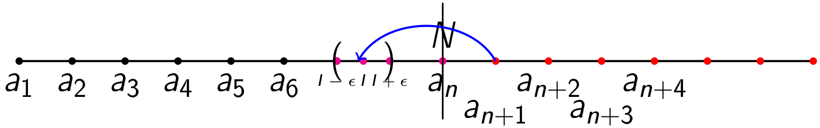

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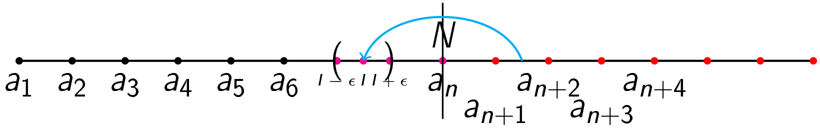

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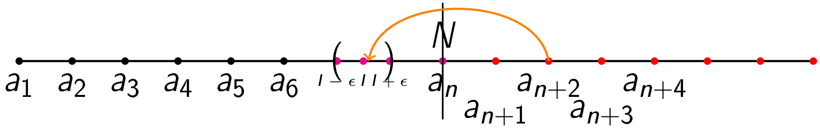

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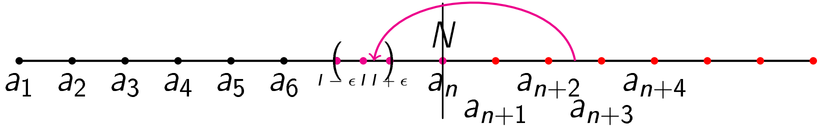

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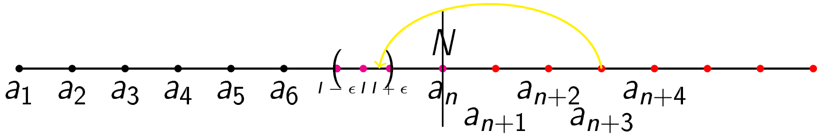

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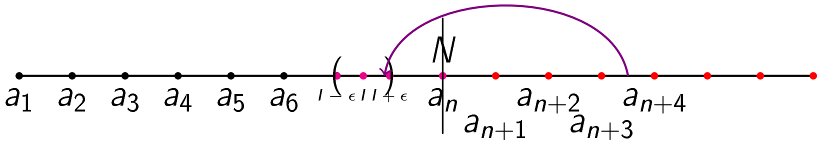

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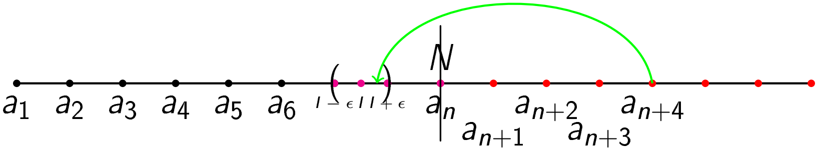

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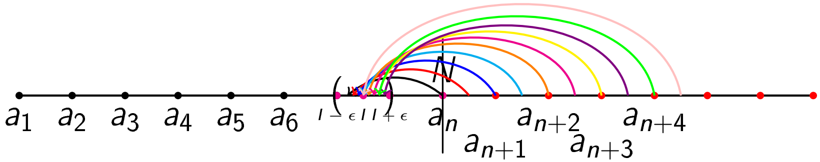

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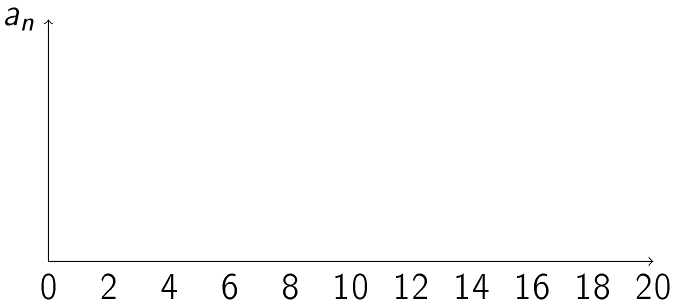
a_n
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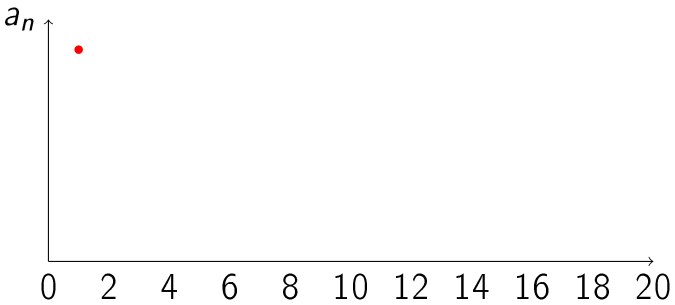
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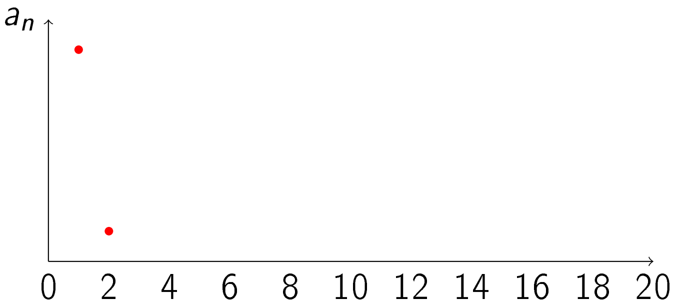

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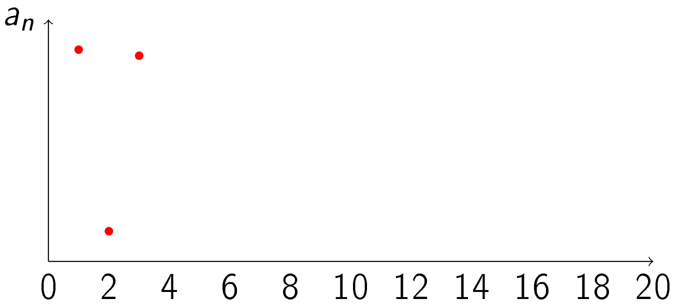
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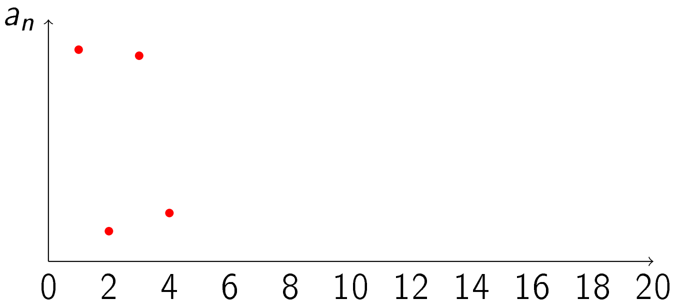
a_n
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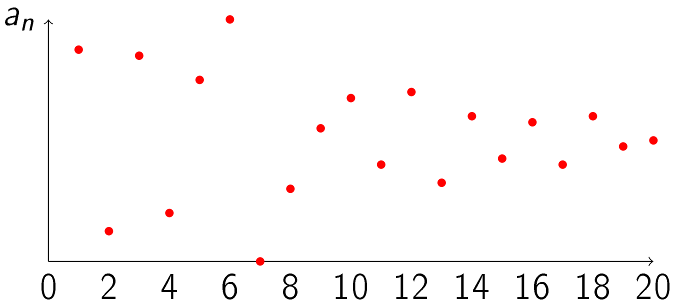
a_n
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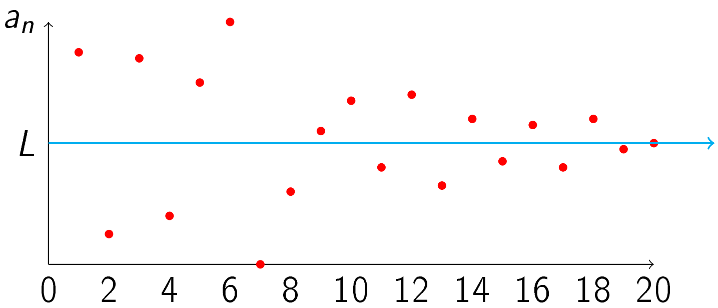
a_n
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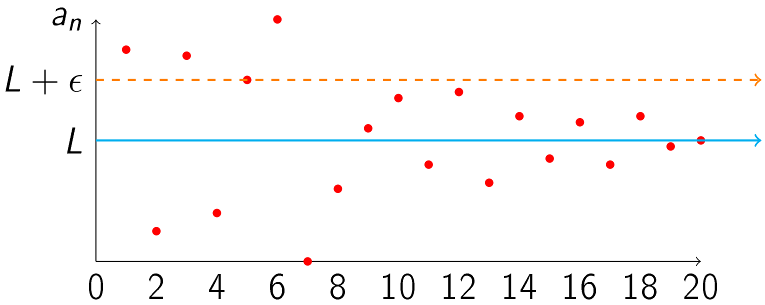
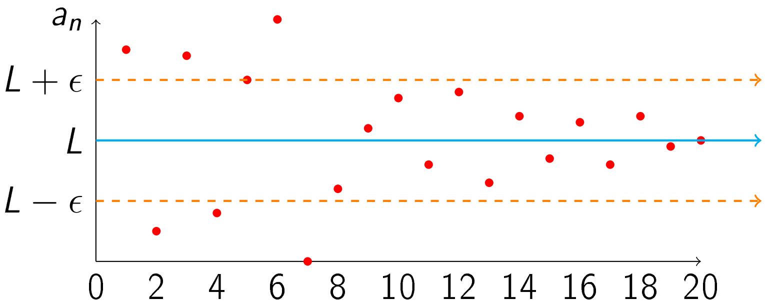
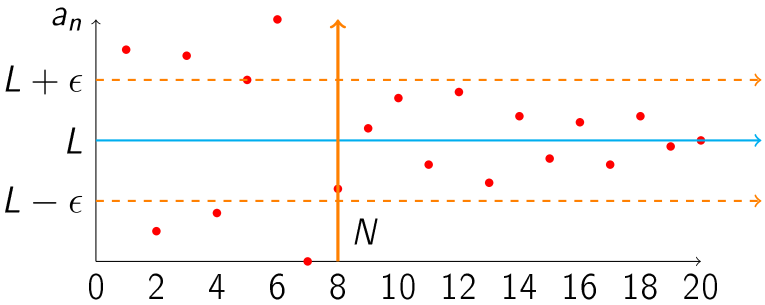
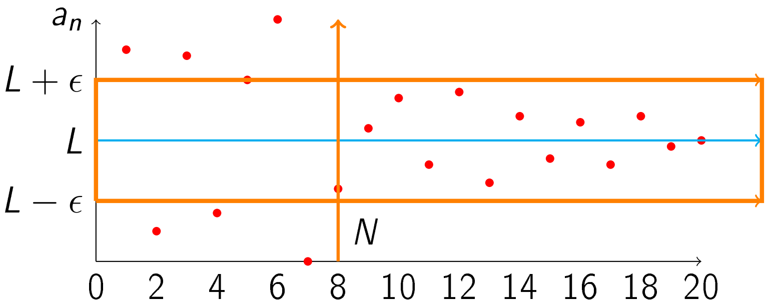
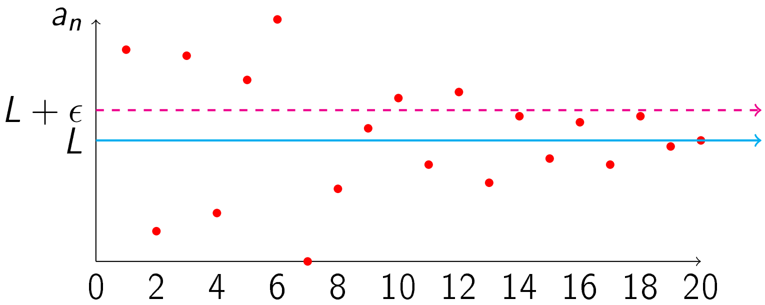

a_n
$L+\epsilon$
L
0 2 4 6 8 10 12 14 16 18 20



a_n
$L + \epsilon$
L
$L - \epsilon$
0 2 4 6 8 10 12 14 16 18 20


a_n
$L + \epsilon$
L
$L - \epsilon$
N
0 2 4 6 8 10 12 14 16 18 20


a_n
$L+\epsilon$
L
$L-\epsilon$
N
0 2 4 6 8 10 12 14 16 18 20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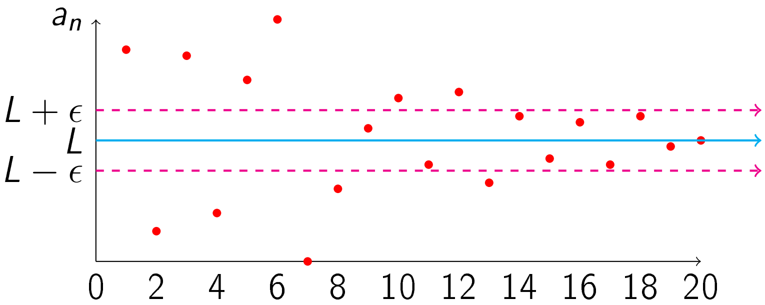
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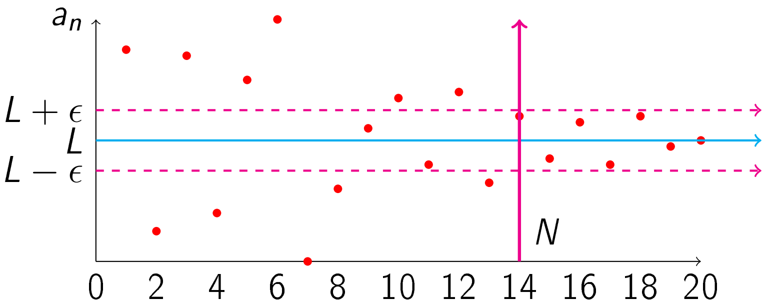

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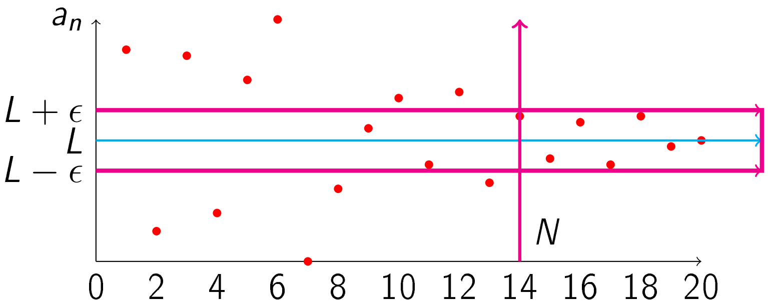
a_n
$L + \epsilon$
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N
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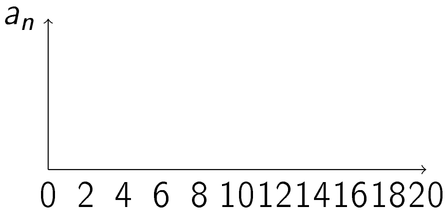
Graphical View



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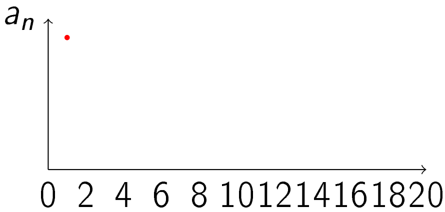
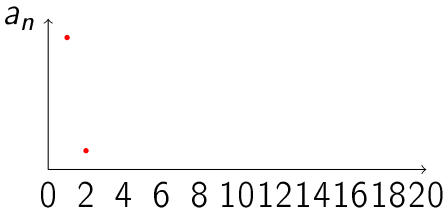
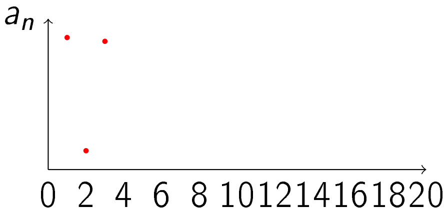


Graphical View



Graphical View



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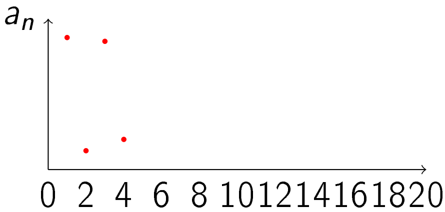

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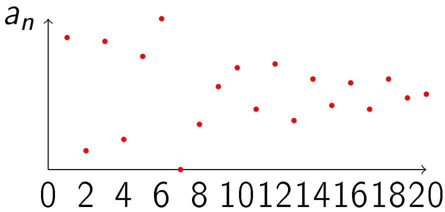

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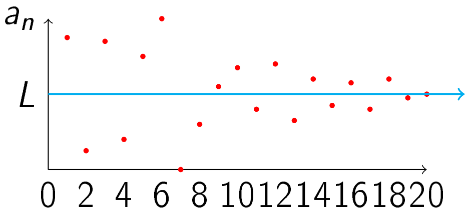

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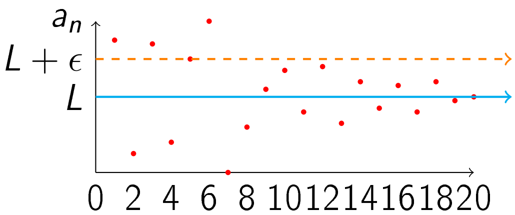
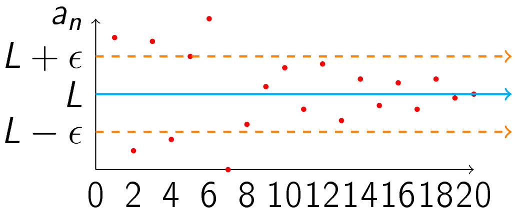


For any $\epsilon > 0$,

Graphical View



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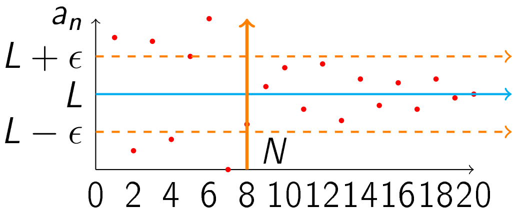
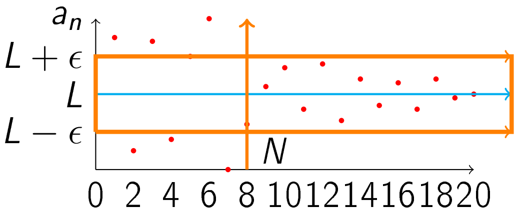


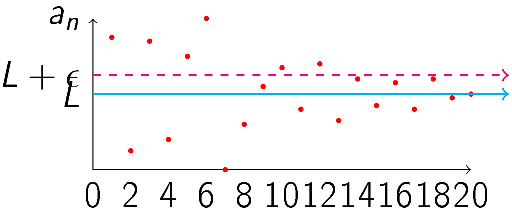
For any $\epsilon > 0$, $\exists$ a positive integer N

Graphical View



For any $\epsilon > 0$, $\exists$ a positive integer N such that $|a_n - L| \leq \epsilon$ for all $n > m$.

Graphical View



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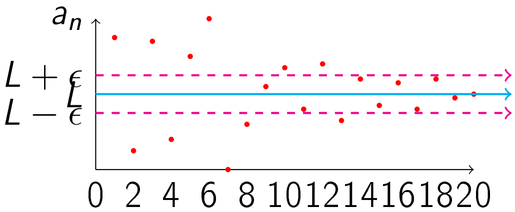

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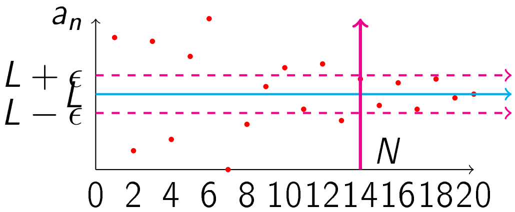

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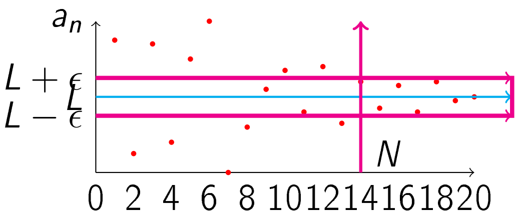

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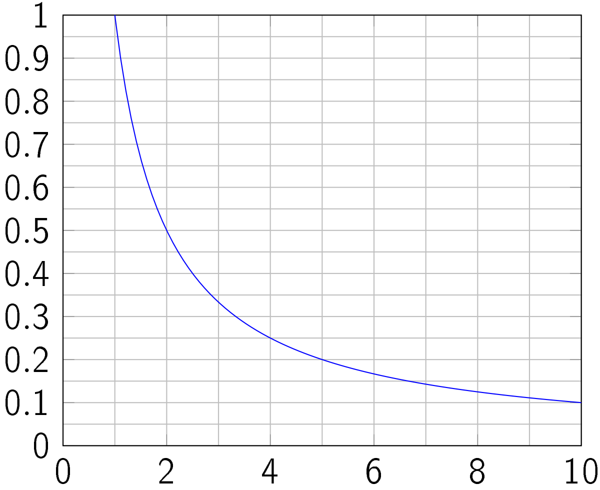
Convergence of the sequence $1/n$



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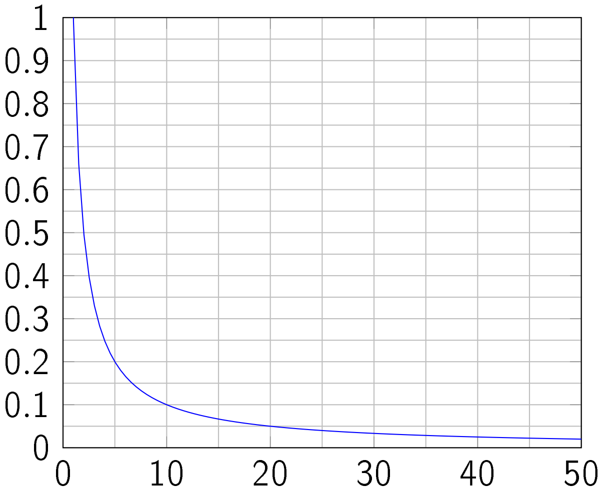

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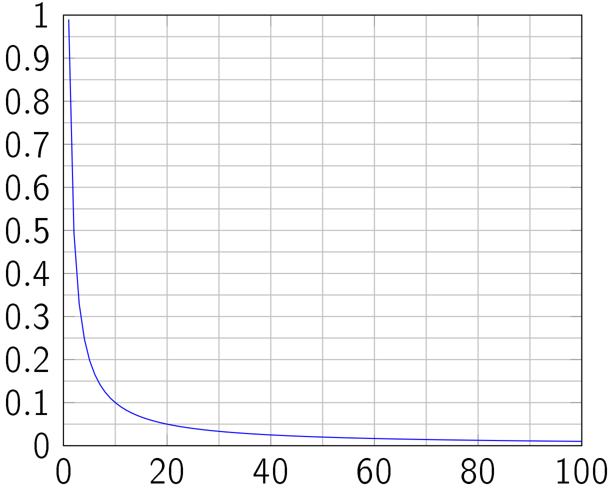
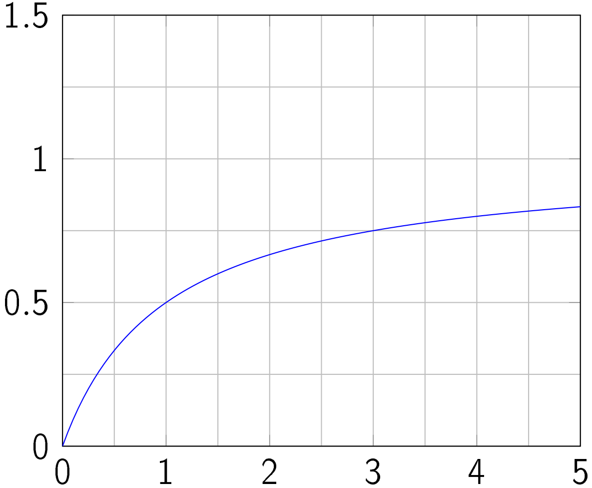


The sequence $\dfrac{n}{n+1}$



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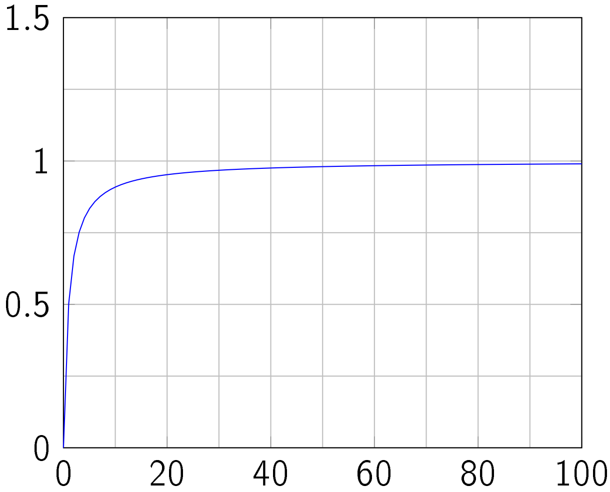

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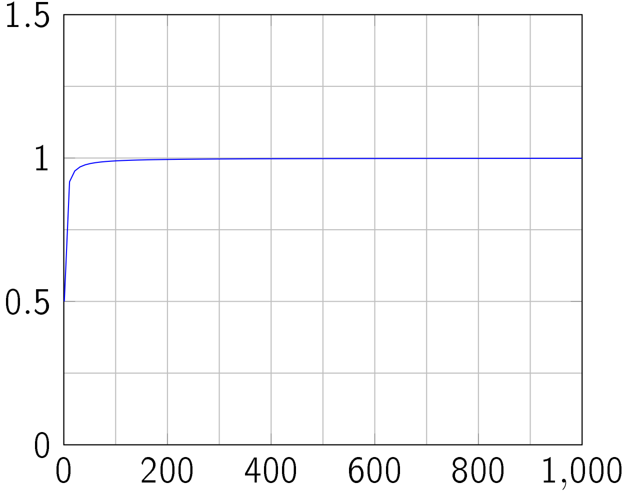
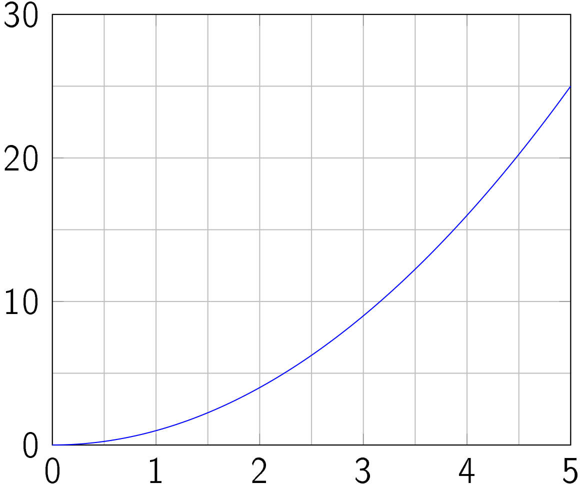


The sequence n^2



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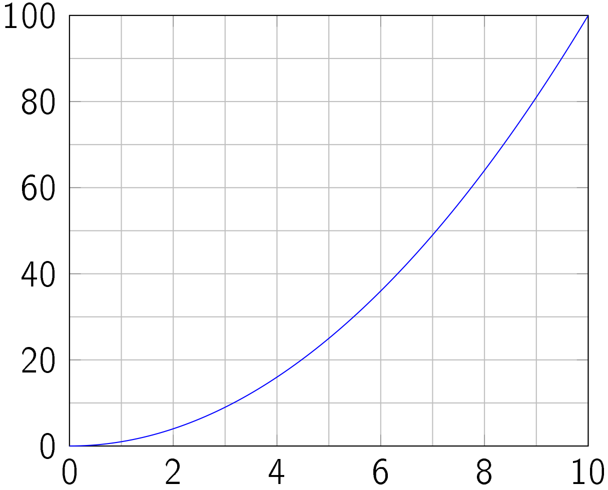

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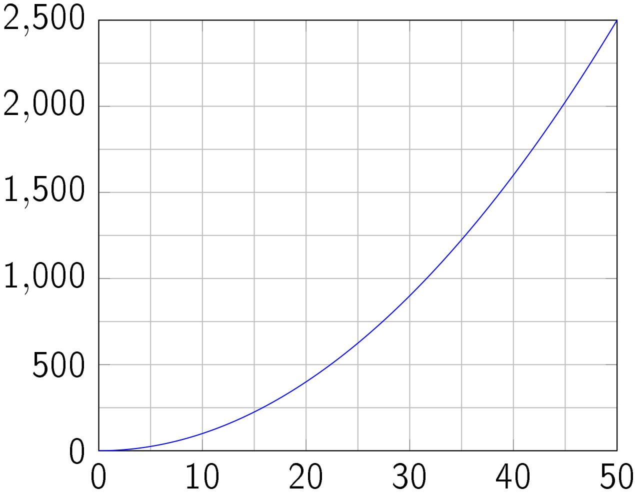

Properties of sequence

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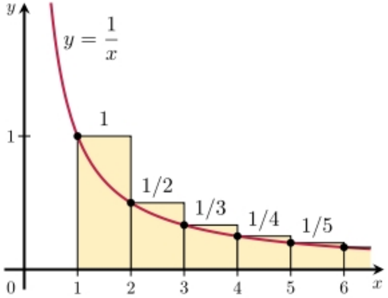
$5, 10, 15, 20, 25, \cdots$	Multiples of 5
$1, 4, 9, 16, 25, \cdots$	Square numbers

Bounded above

A sequence $\{a_n\}$ is said to be bounded above if there exists a real number k such that $a_n \leq k$ for all $n \in \mathbb{N}$. Then k is called the upper bound of the sequence $\{a_n\}$.

Graphical View



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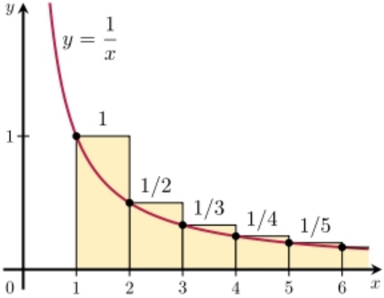

Range of the sequence is $\{1, \frac{1}{2}, \frac{1}{3}, \cdots, \frac{1}{n}\}$,

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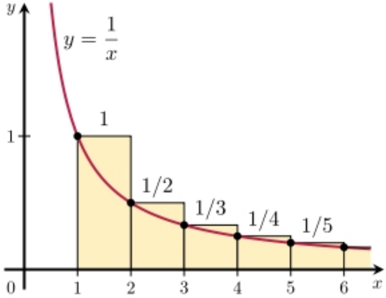
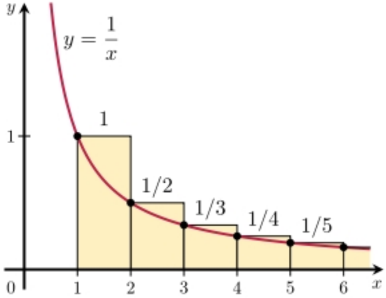


Range of the sequence is $\{1, \frac{1}{2}, \frac{1}{3}, \cdots, \frac{1}{n}\}$, Upper bounds are 1,2,3,$\cdots$

Bounded below

A sequence $\{a_n\}$ is said to be bounded below if there exists a real number k such that $a_n \geq k$ for all $n \in \mathbb{N}$. Then k is called the lower bound of the sequence $\{a_n\}$.

Graphical View



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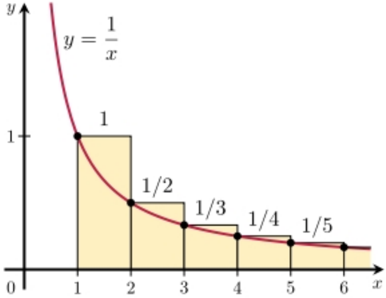

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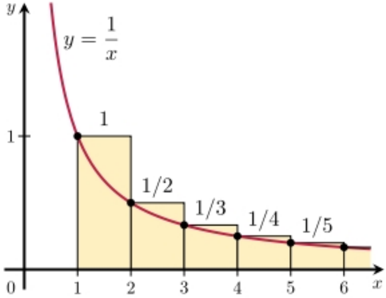

Range of the sequence is $\{1, \frac{1}{2}, \frac{1}{3}, \cdots, \frac{1}{n}\}$, Lower bounds are $0, -1, -2, \cdots, -n$

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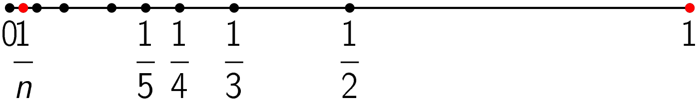

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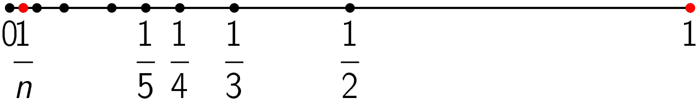

This sequence has both upper and lower bound so it is bounded sequence.

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Example of Bounded Sequence

- The sequence $-1, -2, -3, \cdots, -n, \cdots$ is bounded above but not bounded below. -1 is the lub of the sequence.
- $1, -1, 1, -1, \cdots, 1, -1, \cdots$ is bounded sequence. 1 is lub and -1 is the glb of the sequence.
- Any constant sequence is bounded sequence.

Monotonic increasing

A sequence $\{a_n\}$ is said to be monotonic increasing if $a_n \leq a_{n+1}$ for all n.

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A sequence $\{a_n\}$ is said to be monotonic increasing if $a_n \leq a_{n+1}$ for all n. A sequence $\{a_n\}$ is said to be strictly monotonic increasing if $a_n < a_{n+1}$ for all n.

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- $1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, \cdots$ is a monotonic increasing sequence.

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- $1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, \cdots$ is a monotonic increasing sequence.
- $1, 2, 3, 4, 5 \cdots$ is a strictly monotonic increasing sequence.

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Example

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Example

- $1, \frac{1}{2}, \frac{1}{3}, \cdots, \frac{1}{n} \cdots$ is a strictly monotonic decreasing sequence.
- $-1, -1, -2, -2, -3, -3, -4, -4, \cdots$ is a strictly monotonic decreasing sequence.

Oscillating sequence

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The sequence $\{a_n\}$ given by $1, -1, 1, -1, \cdots$ is neither increasing nor decreasing. Hence $\{a_n\}$ is not a monotonic sequence.

Note

A monotonic increasing sequence $\{a_n\}$ is bounded below and a_1 is the glb of the sequence.

Oscillating sequence

The sequence $\{a_n\}$ given by $1, -1, 1, -1, \cdots$ is neither increasing nor decreasing. Hence $\{a_n\}$ is not a monotonic sequence.

Note

A monotonic increasing sequence $\{a_n\}$ is bounded below and a_1 is the glb of the sequence. A monotonic decreasing sequence $\{a_n\}$ is bounded above and a_1 is the lub of the sequence.

❦ ❦ Time to Interact ❦ ❦

Thank You